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## Velocity and pressure distributions characteristics of coal slurry in floatation cyclone

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**Abstract**

The flowing characteristics of coal slurry in floatation cyclone were investigated on the bases of theoretical calculation of velocity and pressure distributions by Navier-Stokes equation by means of reasonable simplification and modification. Thus, the position of multiphase interface of gas-liquid in the cyclone was located. Apart from this, it could be concluded that an air cylinder exists in the cyclone, the volume of which depends on the velocity, pressure of the fed material and the diameter of overflow pipe. The up-flow of jet stream with fine particles is limited by air cylinder, which is of great significance for the separation process in the floatation cyclone. Meanwhile, separation mechanism by floatation cyclone can be elucidated. Besides, it has been found that there are two types of fluid flow in floatation cyclone, inflow and outflow. In the cases of floatation cyclone with diameter of 100 mm, in the zone with the radius more than 31.5 mm, the radial velocity belong to outflow, and vice versa.

**Keywords:** floatation cyclone; velocity and pressure distributions; air cylinder; separation capability

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**1. Introduction**

In recent years, with the development of coal industry and the improvement of the coal mechanized mining technology, the quantity of fine coal has been increasing. Therefore, separation of coal fines had interested many researchers. Coal flotation is the most common technology for the separation of coal slime. Especially according to the implement of our country's strategy in Clean Coal Technology, it is imperative to develop new equipments due to many factors. People have carried on a large amount of research work to the traditional agitated flotation machine, realized that the flotation behaviour of the traditional agitated flotation machine is carried on in the gravity field basically at the same time, the flotation efficiency is influenced by the time that particles and slurry are in flotation trough. Because tiny particles can not get effective sorted, thin mud is carried seriously, longer flotation time requires larger flotation space, the capacity of flotation is limited.

In the past few years, the authors broke through the traditional principle of flotation machine design according to forefathers' experiences, had applied centrifugal force to strength flotation process which is a great break-through to

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flotation for fine body, had obtained the achievement which makes people gratified in flotation. This paper utilized the Navier-Stokes equation to study about the slurry poured in the launder of cyclone, provided theoretical basis for designing flotation cyclone.

## 2. Simplification of the flow equation

In the previous research of cyclone theory, most authors didn't apply Navier-Stokes equation in the study about the launder distribution in cyclone. This paper makes use of this equation to discuss the change of launder, in order to provide theoretical basis for flotation cyclone.

In order to analyze the change of launder in cyclone theoretically, first of all we have to induct and simplify the flow equation, then adopt cylindrical coordinates for Navier-Stokes equation [1]:

r:

$$\begin{aligned} \rho \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} \right) \\ = \rho F_r - \frac{\partial p}{\partial r} + \mu \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r^2} \right) \end{aligned} \quad (1)$$

$\theta$ :

$$\begin{aligned} \rho \left( \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} \right) \\ = \rho F_\theta - \frac{\partial p}{\partial \theta} + \mu \left( \frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2} \right) \end{aligned} \quad (2)$$

z:

$$\begin{aligned} \rho \left( \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) \\ = \rho F_z - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right) \end{aligned} \quad (3)$$

where,  $u_r, u_\theta, u_z$  are the radial, tangential, and axial velocities;  $F_r, F_\theta, F_z$  are unit body force in three directions;  $\rho$  is the density of slurry;  $\mu$  is the viscosity of slurry; p is the pressure of slurry.

The cyclone launder has two regions of flow field, shown in Fig. 1: one is half free vortex movement, the other is forced vortex movement.

The slurry flows steadily:

$$\frac{\partial u_r}{\partial t} = \frac{\partial u_\theta}{\partial t} = \frac{\partial u_z}{\partial t} = 0$$

$$F_\theta = F_z = 0$$

The distribution of velocity is  $u_\theta r^n = C$  (constant)

$$N = 0.5 \sim 0.9^{[2]}$$

$$\frac{\partial u_\theta}{\partial \theta} = \frac{\partial u_z}{\partial z} = 0$$

$$p = p(r), \frac{\partial p}{\partial \theta} = \frac{\partial p}{\partial z} = 0$$

$$u_z = u_z(r), u_r = u_r(r)$$

$$\frac{\partial u_z}{\partial r} = \frac{\partial u_z}{\partial \theta} = 0, \frac{\partial u_r}{\partial \theta} = \frac{\partial u_r}{\partial z} = 0$$

Equation (1) becomes

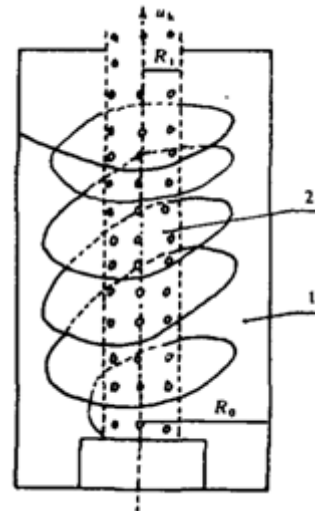


Fig. 1 Flow field region of flotation cyclone  
1—Flow of semi-free vortex; 2—Flow of forced vortex

$$\rho \left( u_r \frac{\partial u_r}{\partial r} - \frac{u_\theta^2}{r} \right) = \rho F_r - \frac{\partial p}{\partial r} + \mu \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) \quad (4)$$

Equation (2) becomes

$$\rho \left( u_r \frac{\partial u_\theta}{\partial r} + \frac{u_r u_\theta}{r} \right) = \mu \left( \frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} \right) \quad (5)$$

Equation (3) becomes

$$\rho u_r \frac{\partial u_z}{\partial r} = \mu \left( \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right) \quad (6)$$

### 3. Theoretical solution to the flow equation

Substitute  $u_\theta r^n = C$  into equation (5), we get the radial velocity distribution in the launder.

$$u_r = -\frac{\mu}{\rho} \frac{(n+1)}{r} \quad (7)$$

Put equation (7) into equation (6), we get

$$\frac{(n+2)}{r} \frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial r^2} = 0 \quad (8)$$

From the differential equation (8), we have

$$u_z = \frac{C_1}{(n+1)} r^{-(n+1)} + C_2 \quad (9)$$

Then the boundary condition is discussed: when the material enters tangentially, it has the tangential velocity only, while the velocities in other directions are zero. When  $r = R_0$  ( $R_0$  is the cyclone overflow radius),  $u_z = 0$ ;  $r = R_1$  ( $R_1$  is the forced vortex radius which is the overflow radius),  $u_z = u_k$  ( $u_k$  is the forced axial velocities which is the overflow velocities).

Put the above two boundary conditions into equation (9), finally we get:

$$u_z = \frac{u_k}{[R_1^{-(n+1)} - R_0^{-(n+1)}] r^{(n+1)}} - \frac{u_k}{[R_1^{-(n+1)} - R_0^{-(n+1)}] R_0^{(n+1)}} \quad (10)$$

Put  $u_\theta = Cr^{-n}$  and equation (7) into equation (4), then we get the distribution of pressure

$$\frac{dp}{dr} = \frac{\mu^2 (n+1)^2}{\rho} \frac{1}{r^3} + \frac{\rho C^2}{r^{(2n+1)}} + \rho F_r \quad (11)$$

$$F_r = \frac{C_2}{r^{(2n+1)}}$$

Integral equation (11), then extract

$$p = -\frac{\mu^2 (n+1)^2}{2\rho} \frac{1}{r^2} - \frac{\rho C^2}{2r^{2n}} + C' \quad (12)$$

When  $P = P_0$  ( $P_0$  is the pressure when the material enters the cyclone),  $r = R_0$ ; then  $u_\theta = u_0$  (the preliminary when entering),  $C = u_0 R_0^n$ , then

$$p = -\frac{\mu^2 (n+1)^2}{2\rho} \frac{1}{r^2} - \frac{\rho u_0^2 R_0^{2n}}{2r^{2n}} + \frac{\mu^2 (n+1)^2}{2\rho} \frac{1}{R_0^2} + \frac{\rho u_0^2}{n} + p_0 \quad (13)$$

When  $p=0$ , substitute the numerical value of all parameters into equation (13) [3], we get  $r = r_0$ .  $r_0$  is the numerical value of the interface. Assuming that

$$u_r = -\frac{\mu}{\rho} \frac{(n+1)}{r} + C_3$$

Let  $u_r = 0$  when  $r = r_0$ , then find the position of contact surface, and let  $u_r$  satisfy the boundary conditions. We get

$$C_3 = \frac{\mu}{\rho} \frac{(n+1)}{r_0} \quad u_r = \frac{\mu}{\rho} \frac{(n+1)}{r_0} - \frac{\mu}{\rho} \frac{(n+1)}{r} \quad (14)$$

Apply the same principle to revise equation (10). Let  $r = r_0, u_z = 0$ , then

$$u_z = \frac{u_k}{[R_1^{-(n+1)} - R_0^{-(n+1)}]r^{(n+1)}} - \frac{u_k}{[R_1^{-(n+1)} - R_0^{-(n+1)}]R_0^{(n+1)}} + C_4$$

$$C_4 = \frac{u_k}{[R_1^{-(n+1)} - R_0^{-(n+1)}]R_0^{(n+1)}} - \frac{u_k}{[R_1^{-(n+1)} - R_0^{-(n+1)}]r_0^{(n+1)}}$$

$$u_z = \frac{u_k}{[R_1^{-(n+1)} - R_0^{-(n+1)}]r^{(n+1)}} - \frac{u_k}{[R_1^{-(n+1)} - R_0^{-(n+1)}]r_0^{(n+1)}} \quad (15)$$

Solving equation (13), (14) and (15), the

changing of  $p, u_r, u_z$  with  $r$  could be obtained.

#### 4. Results and discussions

The changing of  $u_r, u_z$  and  $p$  is calculated by using Quick BASIC language [3]. The distribution curve of change in the flow field is drawn. Calculation is done for cyclone radius of 100 mm and 200 mm, respectively.

##### 4.1. Radial velocity distribution

The distribution of flow field in the cyclone of diameter 100 mm and 200 mm can be seen from Fig. 2 and Fig. 3. At point N in the cyclone, the velocity  $u_r = 0$ . This point is called the dividing point, or the zero interface for the cylindrical cyclone. In other words, the radial velocity includes introversion and extroversion flow in the cyclone. For a cyclone with diameter 100 mm, the radial velocity is extroversion when the cyclone radius is bigger than 31.5 mm, while it is introversion when the cyclone radius is smaller than 31.5 mm.

##### 4.2. Axial velocity distribution

It can be seen from Fig. 2 and Fig. 3, axial velocity decreases as the radius decreases. There is a fragment surface called interface where  $u_z = 0$ . Inside the interface is upward current (overflow), outside the interface is downward current (underflow). The position of interface is related to initial velocity, pressure and diameter of overflow. The initial velocity is the key factor.

##### 4.3. Pressure distribution

It can be seen from Fig. 2 and Fig. 3, pressure increases as the radius increases near the wall of cyclone. But the pressure inside the cyclone decreases as the radius decreases. There is a point, where  $p=0$ , called the pressure dividing point (or interface). This indicates that the pressure near the inner axes of cyclone is negative, and there is

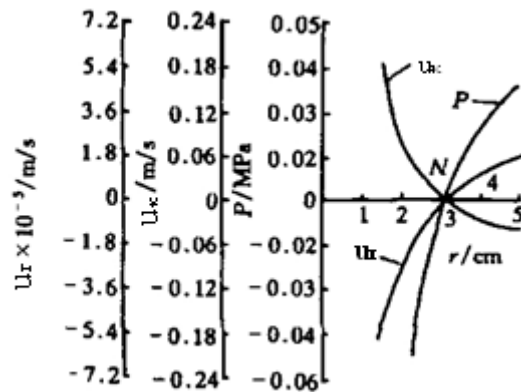


Fig. 2 Velocity and pressure distributions for the diameter 100 mm of floatation cyclone

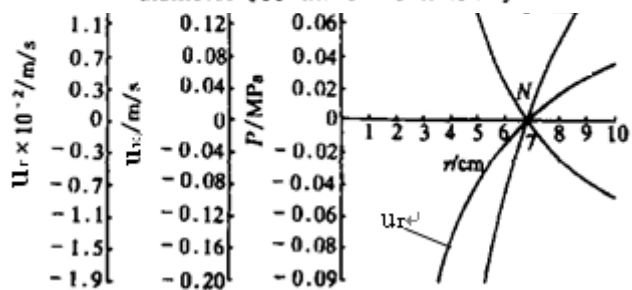


Fig. 3 Velocity and pressure distributions for the diameter 200 mm of floatation cyclone

an air column there. The air column is determined by the initial velocity, pressure and diameter of overflow. The air column restricts the upward current inside, thus its existence has extremely vital significance to separation process.

#### *4.4. Theoretical calculation explanation*

From the domestic and foreign research on cyclone, we can see that almost no scholars use Navier-Stokes equation to analyze the distribution of flow field. There must be some errors as we have made some hypothesis in the process of inducing, it needs to be improved. While the calculated results accord with the distribution of the launder [2,4], meanwhile it explains the mechanism of flotation completely.

### **5. Conclusions**

- (1) The distribution of flow field in flotation cyclone is induced by using Navier-Stokes equation. The position of interface is determined by calculating.
- (2) There exists air column in the centre of cyclone. The air column is determined by initial velocity, pressure and diameter of overflow. Meanwhile it explains the mechanism of flotation completely. The air column restricts the upward current inside, thus its existence has extremely vital significance to separation process.

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